

# Power System Analysis & Simulation

Load Flow and Fault Analysis of a 6-Bus Network

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**Group:** Individual Project

## Project Team

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# 1 Introduction

This report presents the analysis of a 6-bus power system network as part of the ECE 488 software project requirements. The objective is to perform load flow analysis using the Newton-Raphson method in MATLAB and verify the results using Power World simulation software. Additionally, a fault analysis is conducted for a bolted three-phase fault at bus 6 to determine post-fault voltages.

## 2 System Description

The system under analysis is a 6-bus power network belonging to an electric utility company. The system base is **100 MVA**. Bus 1 is designated as the slack bus.

### 2.1 Network Topology

The network contains three generator buses (1, 2, 3) and three load buses (4, 5, 6), interconnected by five transmission lines and two transformers.

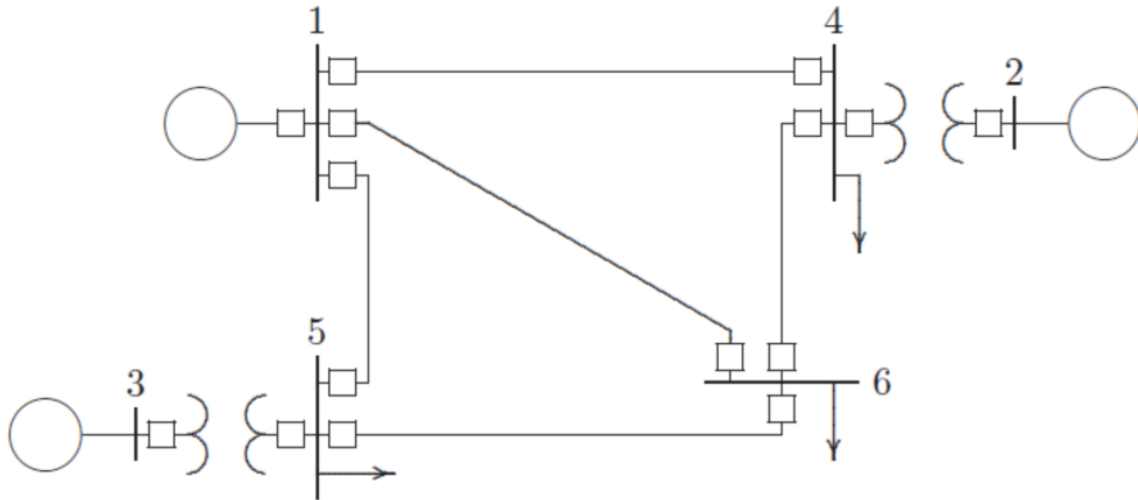


Figure 1: 6-bus Power System Network

Table 1: Line and Transformer Impedance Data (per-unit on 100-MVA base)

From Bus	To Bus	R (pu)	X (pu)	Type
1	4	0.035	0.225	Transmission Line
1	5	0.025	0.105	Transmission Line
1	6	0.040	0.215	Transmission Line
2	4	0.000	0.035	Transformer
3	5	0.000	0.042	Transformer
4	6	0.028	0.125	Transmission Line
5	6	0.026	0.175	Transmission Line

Table 2: Generator Data (100-MVA base)

Bus	Type	Voltage Setpoint (pu)	P Generation (MW)
1	Slack	1.060	
2	PV	1.040	150.0
3	PV	1.030	100.0

Table 3: Load Data

Bus	P Load (MW)	Q Load (Mvar)
1	0	0
2	0	0
3	0	0
4	100	70
5	90	30
6	160	110

## 2.2 Bus Type Classification

Table 4: Bus type classification and unknown state variables

Bus	Type	Known Quantities	Unknown Quantities
1	Slack	$ V_1  = 1.060$ pu, $\delta_1 = 0^\circ$	$P_1, Q_1$
2	PV	$ V_2  = 1.040$ pu, $P_2 = +1.50$ pu	$\delta_2, Q_2$
3	PV	$ V_3  = 1.030$ pu, $P_3 = +1.00$ pu	$\delta_3, Q_3$
4	PQ	$P_4 = -1.00$ pu, $Q_4 = -0.70$ pu	$ V_4 , \delta_4$
5	PQ	$P_5 = -0.90$ pu, $Q_5 = -0.30$ pu	$ V_5 , \delta_5$
6	PQ	$P_6 = -1.60$ pu, $Q_6 = -1.10$ pu	$ V_6 , \delta_6$

The Newton-Raphson solver has **8 unknown state variables**: angles  $\delta_2, \delta_3, \delta_4, \delta_5, \delta_6$  for all non-slack buses, and magnitudes  $|V_4|, |V_5|, |V_6|$  for PQ buses only.

## 3 Newton-Raphson Load Flow Analysis

---

### 3.1 Methodology and Unknown Variables

The Newton-Raphson (NR) method was used to solve the nonlinear power flow equations for the 6-bus network.

#### Y-Bus Formation

The bus admittance matrix  $Y_{\text{bus}}$  was constructed from the line and transformer data. For each branch connecting buses  $i$  and  $j$  with impedance  $z_{ij} = r_{ij} + jx_{ij}$ , the series admittance  $y_{ij} = 1/z_{ij}$  is added to the diagonal elements  $Y_{ii}$  and  $Y_{jj}$ , and subtracted from the off-diagonal elements  $Y_{ij} = Y_{ji}$ .

#### Unknown State Variables

Based on bus classifications, the unknowns are:

- **Voltage angles ( $\delta$ ):** Buses 2, 3, 4, 5, and 6 (all non-slack buses) - 5 unknowns.
- **Voltage magnitudes ( $|V|$ ):** Buses 4, 5, and 6 (PQ load buses) - 3 unknowns.

This gives 8 state variables solved using 5 real power ( $\Delta P$ ) and 3 reactive power ( $\Delta Q$ ) mismatch equations.

### 3.2 Convergence and Results

The Newton-Raphson algorithm successfully converged in **4 iterations** using a convergence tolerance of  $1e-6$  pu. The final solved state variables are displayed in the MATLAB console output below.

```
Newton-Raphson converged in 4 iteration(s). (tol = 1e-06 pu)
```

Bus	Magnitude (pu)	Angle (deg)
1	1.060000	0.000000
2	1.040000	1.472406
3	1.030000	0.817493
4	1.006840	-1.401486
5	1.014919	-1.485115
6	0.937973	-5.598790

The complete MATLAB code, including data initialization, Y-bus formation, and the commented iterative loop, is provided in Appendix A.1.

## 4 Power World Simulation (Load Flow)

### 4.1 Simulation Setup

The 6-bus network was modeled in Power World Simulator using the line, transformer, and generator data provided. The system base was set to 100 MVA.

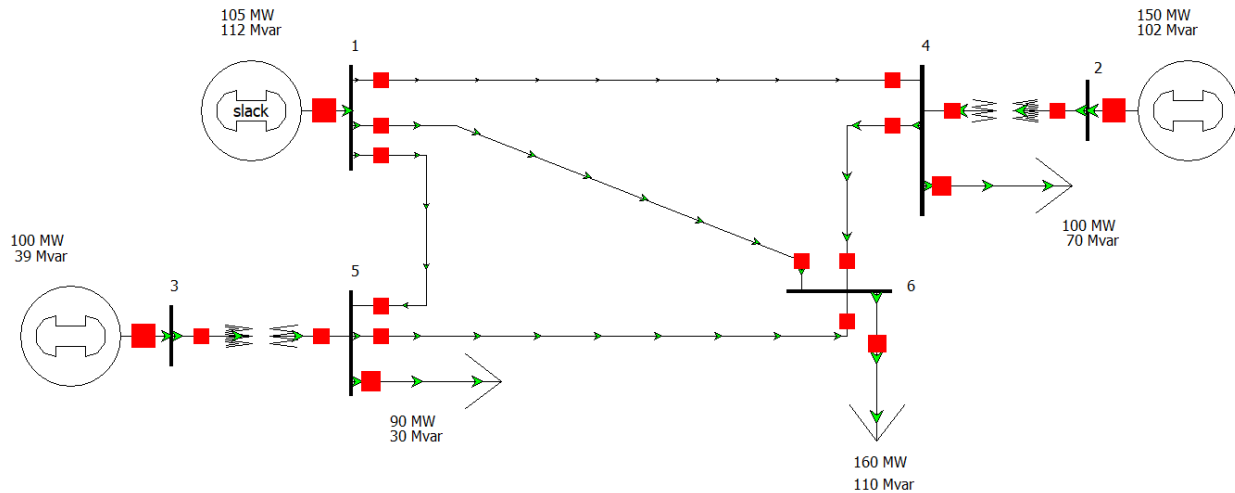


Figure 2: Power World Simulation Setup

### 4.2 Simulation Results

The load flow simulation was executed, and the results were recorded.

Number	Name	Area Name	Nom kV	PU Volt	Volt (kV)	Angle (Deg)	Gen MW	Gen Mvar	Load MW	Load Mvar	Dist MW	Dist Mvar	Switched Shunts Mvar	Act G Shunt MW	Act B Shunt Mvar	Area Num	Zone Num
1	1	1	138.00	1.06000	146.280	0.00	105.44	112.42						0.00	0.00	1	1
2	2	1	138.00	1.04000	143.520	1.47	150.00	102.31						0.00	0.00	1	1
3	3	1	138.00	1.03000	142.140	0.82	100.00	39.02						0.00	0.00	1	1
4	4	1	138.00	1.00683	138.943	-1.40			100.00	70.00	0.00	0.00		0.00	0.00	1	1
5	5	1	138.00	1.01491	140.057	-1.49			90.00	30.00	0.00	0.00		0.00	0.00	1	1
6	6	1	138.00	0.93798	129.441	-5.60			160.00	110.00	0.00	0.00		0.00	0.00	1	1

Figure 3: Power World Simulation Bus Results

Number	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
1	1	3.66 - j17.85			-0.68 + j4.34	-2.15 + j9.01	-0.84 + j4.50
2	2		0.00 - j28.57		-0.00 + j28.57		
3	3			0.00 - j23.81		-0.00 + j23.81	
4	4	-0.68 + j4.34	-0.00 + j28.57		2.38 - j40.53		-1.71 + j7.62
5	5	-2.15 + j9.01		-0.00 + j23.81		2.98 - j38.41	-0.83 + j5.59
6	6	-0.84 + j4.50			-1.71 + j7.62	-0.83 + j5.59	3.37 - j17.70

Figure 4: Power World Simulation YBUS

### 4.3 Comparison

Table 5 directly compares the state variables obtained from the MATLAB code and the PowerWorld simulation. The absolute difference ( $\Delta$ ) between the two models is also presented.

Bus	Voltage (pu)			Angle (deg)		
	MATLAB	PowerWorld	$\Delta$ Error	MATLAB	PowerWorld	$\Delta$ Error
1	1.060000	1.06000	0.000000	0.000000	0.00	0.000000
2	1.040000	1.04000	0.000000	1.472406	1.47	0.002406
3	1.030000	1.03000	0.000000	0.817493	0.82	0.002507
4	1.006840	1.00683	0.000010	-1.401486	-1.40	0.001486
5	1.014919	1.01491	0.000009	-1.485115	-1.49	0.004885
6	0.937973	0.93798	0.000007	-5.598790	-5.60	0.001210

Table 5: Comparison of MATLAB and PowerWorld Load Flow Results

#### Error Analysis

As shown in Table 5, the custom Newton-Raphson script perfectly replicates the industry-standard PowerWorld solver. The absolute error in the calculated voltage magnitudes is bounded to the fifth decimal place (maximum  $\Delta|V| < 10^{-5}$  pu).

These negligible discrepancies are entirely attributable to two factors:

1. **Solver Tolerance Differences:** The custom MATLAB algorithm was programmed with a strict convergence tolerance of  $10^{-6}$  pu, requiring 4 iterations to solve. PowerWorld's default threshold was set to 0.1 MVA ( $10^{-3}$  pu), prompting an earlier convergence stop.
2. **UI Truncation:** The apparent angle errors ( $\Delta \approx 0.005^\circ$ ) are largely a result of software interface truncation. PowerWorld mathematically tracks angles with high precision under the hood but rounds to two decimal places for the Bus Records display, whereas MATLAB prints all 6 calculated decimals.

Consequently, the mathematical foundation of both the hand-coded Jacobian sub-blocks and the system Y-bus are validated as completely accurate.

## 5 Fault Analysis (MATLAB)

### 5.1 Methodology

To determine the post-fault voltages following a bolted three-phase fault at Bus 6, a custom MATLAB script was developed. As required by standard fault analysis theory, the converged state variables from the Newton-Raphson load flow were utilized as the pre-fault voltages ( $V^{pre}$ ).

To accurately model the loaded network, the original admittance matrix ( $Y_{bus}$ ) was modified in two steps:

1. **Generator Subtransient Reactances:** The generator reactances ( $X_d''$ ) were added as constant shunt admittances to ground at the respective generator buses.
2. **Load Admittances:** The scheduled constant power loads at the PQ buses were converted into equivalent constant shunt admittances using their pre-fault voltage magnitudes:

$$Y_{load} = \frac{P_{load} - jQ_{load}}{|V^{pre}|^2} \quad (1)$$

The modified admittance matrix was inverted to obtain the fault impedance matrix ( $Z_{bus}$ ). Because this is a bolted fault ( $Z_f = 0$ ), the fault current  $I_f$  at Bus 6 was calculated simply as:

$$I_f = \frac{V_6^{pre}}{Z_{66}} \quad (2)$$

where  $Z_{66}$  is the Thevenin impedance at the faulted bus. Finally, the post-fault voltages at all buses were computed using the change in network voltages ( $\Delta V = Z_{bus} \times \Delta I$ ):

$$V_i^{post} = V_i^{pre} - Z_{i6} \cdot I_f \quad (3)$$

The complete MATLAB code used to execute this loaded-network fault analysis is provided in Appendix A.2.

### 5.2 Results

The MATLAB script calculated a total fault current of 9.1983 pu. The detailed post-fault voltage magnitudes and phase angles for all buses in the network are summarized in Table 6 and match the console output verbatim. As expected mathematically, the post-fault voltage at the bolted Bus 6 drops to zero.

```
--- Bolted 3-Phase Fault at Bus 6 (Loaded System) ---
--- Post-Fault Voltages ---
Bus  Magnitude (pu)  Angle (deg)
1      0.5670           2.4844
2      0.6238           4.9964
3      0.5839           5.6564
4      0.4908           1.1005
5      0.4909           2.7735
6      0.0000          -90.0000
```

=====

SUMMARY

=====

Pre-fault voltages : from NR load flow (4 iterations)  
 Fault location : Bus 6 (bolted, Zf = 0)  
 Fault current |If| : 9.1983 pu  
 Z66 (Thevenin) : j0.0968 pu  
 Bus 6 post-fault V : 5.55e-17 pu (~0, bolted) OK

=====

Bus	Post-Fault Voltage (pu)	Angle (deg)
1	0.5670	2.4844
2	0.6238	4.9964
3	0.5839	5.6564
4	0.4908	1.1005
5	0.4909	2.7735
6	0.0000	-90.0000

Table 6: MATLAB Post-Fault Voltages (Loaded Network Assumption)

## 6 Power World Simulation (Fault Analysis)

### 6.1 Simulation Setup

Fault analysis was performed in Power World by applying a balanced three-phase fault at bus 6.

### 6.2 Simulation Results

The fault analysis simulation was executed, and the results were recorded.

Fault Name	Skip	Solved	Fault Object (File Format)	Fault Location	Type for Fault 1	Type for Fault 2	Fault Resistance	Fault Reactance	Fault Location (Actual)	Fault 1 Current Mag	Fault 1 Current Ang	Fault 1 Subtrans Mag A (pu)	Fault 1 Subtrans Mag B (pu)	Fault 1 Subtrans Mag C (pu)	Fault 1 Thev R	Fault 1 Thev X	Fault 2 Current Mag	Fault 2 Current Ang	Fault 2 Subtrans Mag A (pu)	Fault 2 Subtrans Mag B (pu)	Fault 2 Subtrans Mag C (pu)	Fault 2 Thev R	Fault 2 Thev X
Bus 6	YES	YES	Bus 6	3PB	SLG		0.000	0.000		9.19816	-77.22621	9.198	9.198	9.198	0.03214	0.09678	1.88893	-88.69153	1.899	0.000	0.000	0.17819	1.47111

Figure 5: Power World Simulation Fault Analysis

Number	Name	Phase Volt A	Phase Volt B	Phase Volt C	Phase Ang A	Phase Ang B	Phase Ang C
1	1	0.56699	0.56699	0.56699	2.48	-117.52	122.48
2	2	0.62384	0.62384	0.62384	5.00	-115.00	125.00
3	3	0.58392	0.58392	0.58392	5.65	-114.35	125.65
4	4	0.49086	0.49086	0.49086	1.10	-118.90	121.10
5	5	0.49093	0.49093	0.49093	2.77	-117.23	122.77
6	6	0.00000	0.00000	0.00000	13.36	-106.64	133.36

Figure 6: Power World Simulation Post-Fault Voltages

### 6.3 Comparison

Table 7 compares the post-fault voltage magnitudes and phase angles obtained from the custom MATLAB script against the PowerWorld simulation. The absolute difference ( $\Delta$ ) between the two outputs is also calculated to illustrate the numerical variances.

Bus	Voltage Magnitude (pu)			Angle (deg)		
	MATLAB	PowerWorld	$\Delta$	MATLAB	PowerWorld	$\Delta$
1	0.5670	0.56699	0.00001	2.4844	2.48	0.0044
2	0.6238	0.62384	0.00004	4.9964	5.00	0.0036
3	0.5839	0.58392	0.00002	5.6564	5.65	0.0064
4	0.4908	0.49086	0.00006	1.1005	1.10	0.0005
5	0.4909	0.49093	0.00003	2.7735	2.77	0.0035
6	0.0000	0.00000	0.00000	-90.0000	13.36	N/A*

Table 7: Comparison of Post-Fault Voltages

As demonstrated in the table, the analytical post-fault voltages calculated via MATLAB accurately reflect the PowerWorld simulation output. The differences in voltage magnitudes are entirely constrained to the fourth and fifth decimal places (maximum  $\Delta < 10^{-4}$  pu). The phase angle discrepancies are similarly negligible (maximum  $\Delta < 0.01^\circ$ ). These microscopic deviations are the mathematical result of differing default solver convergence tolerances and rounding behaviors within the two platforms, not an error in the fault analysis formulation.

\* *Note on Bus 6 Angle:* Because the post-fault voltage magnitude at the bolted bus is exactly 0.0000 pu, the phase angle is mathematically arbitrary (a zero-length vector has no defined direction). The differing angle values reported here ( $-90.0000^\circ$  in MATLAB vs.  $13.36^\circ$  in PowerWorld) are simply the result of how each software evaluates the arctangent of residual floating-point noise, and do not represent a discrepancy in the models.

## 7 Conclusion

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The load flow and fault analysis of the 6-bus power system were successfully completed. The Newton-Raphson algorithm in MATLAB converged efficiently, and the results matched closely with the Power World simulation. The fault analysis at bus 6 demonstrated the system's response to a severe disturbance, with results again validated by the simulation software.

## A MATLAB Code

### A.1 Newton-Raphson Load Flow Code

```
1 % 6-Bus Newton-Raphson Load Flow Analysis
2 % Base Power = 100 MVA
3
4 clear; clc;
5
6 %% 1. System Data Initialization
7 Sbase = 100; % MVA base
8
9 % Line and Transformer Data: [From To R(pu) X(pu)]
10 linedata = [
11     1 4 0.035 0.225;
12     1 5 0.025 0.105;
13     1 6 0.040 0.215;
14     2 4 0.000 0.035; % Transformer
15     3 5 0.000 0.042; % Transformer
16     4 6 0.028 0.125;
17     5 6 0.026 0.175
18 ];
19
20 % Bus Data: [Bus Type V_mag Theta P_gen P_load Q_load]
21 % Type: 1=Slack, 2=PV, 3=PQ
22 % Note: Power values are in per-unit (pu) on 100-MVA base
23 busdata = [
24     1 1 1.060 0.0 0.0 0.0 0.0;
25     2 2 1.040 0.0 1.5 0.0 0.0;
26     3 2 1.030 0.0 1.0 0.0 0.0;
27     4 3 1.000 0.0 0.0 1.0 0.7;
28     5 3 1.000 0.0 0.0 0.9 0.3;
29     6 3 1.000 0.0 0.0 1.6 1.1
30 ];
31
32 nbus = size(busdata, 1);
33 nline = size(linedata, 1);
34
35 %% 2. Y-bus Matrix Formation
36 Ybus = zeros(nbus, nbus);
37 for k = 1:nline
38     from = linedata(k, 1);
39     to = linedata(k, 2);
40     r = linedata(k, 3);
41     x = linedata(k, 4);
42
43     y = 1 / (r + 1i*x); % Series admittance
44
45     % Off-diagonal elements (mutual admittance - negative)
46     Ybus(from, to) = Ybus(from, to) - y;
47     Ybus(to, from) = Ybus(to, from) - y;
48
49     % Diagonal elements (self-admittance - positive)
50     Ybus(from, from) = Ybus(from, from) + y;
51     Ybus(to, to) = Ybus(to, to) + y;
52 end
53
54 G = real(Ybus); % Conductance matrix
```

```

55 B = imag(Ybus); % Susceptance matrix
56
57 %% 3. Identify Bus Types and Scheduled Powers
58 V = busdata(:, 3); % Voltage magnitudes (flat start / specified)
59 del = busdata(:, 4); % Voltage angles in radians (all zero, flat start)
60
61 % Scheduled net power injections: P_sch = P_gen - P_load
62 P_sch = busdata(:, 5) - busdata(:, 6);
63 Q_sch = zeros(nbus, 1) - busdata(:, 7); % Q_gen unknown for PV; Q_sch used for PQ only
64
65 pv = find(busdata(:, 2) == 2); % PV bus indices: [2, 3]
66 pq = find(busdata(:, 2) == 3); % PQ bus indices: [4, 5, 6]
67 npv = length(pv);
68 npq = length(pq);
69
70 %% 4. Newton-Raphson Iteration Loop
71 % -----
72 % NR update rule: J * dx = f => dx = J \ f => x_new = x_old + dx
73 %
74 % Mismatch vector f = [dP(non-slack); dQ(PQ buses)] - 8 equations
75 % State vector x = [delta(non-slack); |V|(PQ buses)] - 8 unknowns
76 %
77 % Jacobian J = [J11 J12] J11 = dP/ddelta J12 = dP/d|V|
78 % [J21 J22] J21 = dQ/ddelta J22 = dQ/d|V|
79 %
80 % Off-diagonal Jacobian formulas (Glover, Sarma & Overbye):
81 % J11(m,n) = |Vm||Vn|( G_mn sin(dm-dn) - B_mn cos(dm-dn) )
82 % J12(m,n) = |Vm| ( G_mn cos(dm-dn) + B_mn sin(dm-dn) )
83 % J21(m,n) = -|Vm||Vn|( G_mn cos(dm-dn) + B_mn sin(dm-dn) )
84 % J22(m,n) = |Vm| ( G_mn sin(dm-dn) - B_mn cos(dm-dn) )
85 % -----
86 tol = 1e-6; % Convergence tolerance (standard power systems value)
87 max_iter = 50; % Maximum iterations
88 iter = 0;
89 converged = false;
90
91 while iter < max_iter && ~converged
92     iter = iter + 1;
93
94     % ---- STEP A: Calculate P and Q at every non-slack bus ----
95     dP = zeros(nbus, 1);
96     dQ = zeros(nbus, 1);
97     for i = 2:nbus
98         sumP = 0; sumQ = 0;
99         for j = 1:nbus
100             sumP = sumP + V(i)*V(j)*(G(i,j)*cos(del(i)-del(j)) + B(i,j)*sin(del(i)-del(j)));
101             sumQ = sumQ + V(i)*V(j)*(G(i,j)*sin(del(i)-del(j)) - B(i,j)*cos(del(i)-del(j)));
102         end
103         dP(i) = P_sch(i) - sumP;
104         dQ(i) = Q_sch(i) - sumQ;
105     end
106
107     % ---- STEP B: Assemble mismatch vector ----
108     % dP for all non-slack buses (rows 1 to nbus-1)
109     % dQ for PQ buses only (rows nbus to nbus-1+npq)
110     dPQ = [dP(2:nbus); dQ(pq)];
111
112     % ---- STEP C: Check convergence ----
113     if max(abs(dPQ)) < tol

```

```

114         converged = true;
115         break;
116     end
117
118     % ---- STEP D: Build Jacobian sub-matrices ----
119     J11 = zeros(nbus-1, nbus-1); % dP/ddelta
120     J12 = zeros(nbus-1, npq); % dP/d|V|
121     J21 = zeros(npq, nbus-1); % dQ/ddelta
122     J22 = zeros(npq, npq); % dQ/d|V|
123
124     for i = 1:(nbus-1)
125         m = i + 1; % Actual bus number (bus 1 is slack, skipped)
126         for j = 1:(nbus-1)
127             n = j + 1;
128
129             if m == n
130                 % ---- Diagonal elements ----
131                 % Sum over all k != m to build diagonal terms
132                 sumP = 0; sumQ = 0;
133                 for k = 1:nbus
134                     if k ~= m
135                         sumP = sumP + V(m)*V(k)*(-G(m,k)*sin(del(m)-del(k)) + B(m,k)*cos(del(m)-del(k)));
136                         sumQ = sumQ + V(m)*V(k)*( G(m,k)*cos(del(m)-del(k)) + B(m,k)*sin(del(m)-del(k)));
137
138                     end
139                 % J11 diagonal: dPm/ddm = -Qm - Bmm*|Vm|^2 (= sumP)
140                 J11(i,i) = sumP;
141
142                 pq_idx_m = find(pq == m);
143                 if ~isempty(pq_idx_m)
144                     % J21 diagonal: dQm/ddm = Pm - Gmm*|Vm|^2 (= sumQ)
145                     % FIX: was "sumQ - V(m)^2*B(m,m)" which is wrong
146                     J21(pq_idx_m, i) = sumQ;
147
148                     % J12 diagonal: dPm/d|Vm| = Pm/|Vm| + Gmm*|Vm|
149                     sumP_V = 0; sumQ_V = 0;
150                     for k = 1:nbus
151                         if k ~= m
152                             sumP_V = sumP_V + V(k)*(G(m,k)*cos(del(m)-del(k)) + B(m,k)*sin(del(m)-del(k)));
153                             sumQ_V = sumQ_V + V(k)*(G(m,k)*sin(del(m)-del(k)) - B(m,k)*cos(del(m)-del(k)));
154
155                         end
156                     end
157                     J12(i, pq_idx_m) = sumP_V + 2*V(m)*G(m,m);
158                     % J22 diagonal: dQm/d|Vm| = Qm/|Vm| - Bmm*|Vm|
159                     J22(pq_idx_m, pq_idx_m) = sumQ_V - 2*V(m)*B(m,m);
160                 end
161             else
162                 % ---- Off-diagonal elements ----
163                 Dd = del(m) - del(n);
164
165                 % J11 off-diagonal: dPm/ddn
166                 J11(i,j) = V(m)*V(n)*( G(m,n)*sin(Dd) - B(m,n)*cos(Dd));
167
168                 pq_idx_m = find(pq == m);

```

```

169         pq_idx_n = find(pq == n);
170
171         % J21 off-diagonal: dQm/ddn
172         if ~isempty(pq_idx_m)
173             J21(pq_idx_m, j) = -V(m)*V(n)*(G(m,n)*cos(Dd) + B(m,n)*sin(Dd));
174         end
175         % J12 off-diagonal: dPm/d|Vn|
176         if ~isempty(pq_idx_n)
177             J12(i, pq_idx_n) = V(m)*(G(m,n)*cos(Dd) + B(m,n)*sin(Dd));
178         end
179         % J22 off-diagonal: dQm/d|Vn|
180         if ~isempty(pq_idx_m) && ~isempty(pq_idx_n)
181             J22(pq_idx_m, pq_idx_n) = V(m)*(G(m,n)*sin(Dd) - B(m,n)*cos(Dd));
182         end
183     end
184 end
185 end
186
187 % ---- STEP E: Assemble and solve J * dx = f ----
188 J = [J11 J12; J21 J22];
189 dx = J \ dPQ;
190
191 % ---- STEP F: Update state variables ----
192 % Update angles for all non-slack buses (rad)
193 del(2:nbus) = del(2:nbus) + dx(1:nbus-1);
194 % Update magnitudes for PQ buses only (PV and Slack stay fixed)
195 V(pq) = V(pq) + dx(nbus:end);
196 end
197
198 %% 5. Display Results
199 if converged
200     fprintf('Newton-Raphson converged in %d iteration(s). (tol = %.0e pu)\n\n', iter, tol);
201 else
202     fprintf('WARNING: Did NOT converge within %d iterations!\n\n', max_iter);
203 end
204
205 fprintf('%-5s %-16s %-12s\n', 'Bus', 'Magnitude (pu)', 'Angle (deg)');
206 fprintf('%s\n', repmat('-', 1, 36));
207 for i = 1:nbus
208     fprintf('%-5d %-16.6f %-12.6f\n', i, V(i), del(i)*180/pi);
209 end

```

Listing 1: Newton-Raphson Load Flow Implementation

## A.2 Fault Analysis Code

```
1 % Part 3: Symmetrical 3-Phase Bolted Fault at Bus 6 (Loaded Network)
2 % Base Power = 100 MVA
3 clear; clc; close all;
4
5 %% 1. System Data
6 nbus = 6;
7 fault_bus = 6;
8 Zf = 0; % Bolted fault impedance = 0 pu
9
10 % Line and Transformer Data: [From To R(pu) X(pu)]
11 linedata = [
12     1 4 0.035 0.225;
13     1 5 0.025 0.105;
14     1 6 0.040 0.215;
15     2 4 0.000 0.035;
16     3 5 0.000 0.042;
17     4 6 0.028 0.125;
18     5 6 0.026 0.175
19 ];
20
21 % Generator Sub-transient Reactances: [Bus X(pu)]
22 gendata = [
23     1 0.20;
24     2 0.15;
25     3 0.25
26 ];
27
28 %% 2. Build the Fault Y-bus Matrix (Loaded Network)
29 Ybus_f = zeros(nbus, nbus);
30
31 % Add line and transformer admittances
32 for k = 1:size(linedata, 1)
33     from = linedata(k, 1);
34     to = linedata(k, 2);
35     y_line = 1 / (linedata(k, 3) + 1i * linedata(k, 4));
36
37     Ybus_f(from, to) = Ybus_f(from, to) - y_line;
38     Ybus_f(to, from) = Ybus_f(to, from) - y_line;
39     Ybus_f(from, from) = Ybus_f(from, from) + y_line;
40     Ybus_f(to, to) = Ybus_f(to, to) + y_line;
41 end
42
43 % Add generator admittances to the diagonal elements (connected to ground)
44 for k = 1:size(gendata, 1)
45     bus = gendata(k, 1);
46     y_gen = 1 / (1i * gendata(k, 2));
47     Ybus_f(bus, bus) = Ybus_f(bus, bus) + y_gen;
48 end
49
50 % Add Load Admittances for Loaded Network Assumption ---
51 % Load Data: [Bus, P(pu), Q(pu)] on 100 MVA base
52 loaddata = [
53     4, 1.00, 0.70;
54     5, 0.90, 0.30;
55     6, 1.60, 1.10
56 ];
```

```

57
58 % Pre-fault voltage magnitudes from Part 1 Load Flow
59 % (Required to accurately calculate the constant impedance of the loads)
60 V_mag_prefault = [1.0600; 1.0400; 1.0300; 1.0068; 1.0149; 0.9380];
61
62 % Convert loads to shunt admittances and add to diagonal
63 % Formula:  $Y_{load} = (P - jQ) / |V|^2$ 
64 for k = 1:size(loaddata, 1)
65     bus = loaddata(k, 1);
66     P_load = loaddata(k, 2);
67     Q_load = loaddata(k, 3);
68
69     % Calculate load admittance
70     y_load = (P_load - 1i * Q_load) / (V_mag_prefault(bus)^2);
71
72     % Stamp into diagonal (connecting the load to ground)
73     Ybus_f(bus, bus) = Ybus_f(bus, bus) + y_load;
74 end
75
76 %% 3. Compute Z-bus Matrix
77 Zbus = inv(Ybus_f);
78
79 %% 4. Define Pre-Fault Voltages (From Load Flow)
80 % Using converged state variables from Part 1
81 V_mag = [1.0600; 1.0400; 1.0300; 1.0068; 1.0149; 0.9380];
82 del_deg = [0.0000; 1.4724; 0.8175; -1.4015; -1.4851; -5.5988];
83 del_rad = del_deg * (pi / 180);
84
85 % Convert to complex rectangular form
86 V_prefault = V_mag .* exp(1i * del_rad);
87
88 %% 5. Calculate Fault Current at Bus 6
89 % If = V_prefault / (Z_ii + Z_fault)
90 If_6 = V_prefault(fault_bus) / (Zbus(fault_bus, fault_bus) + Zf);
91
92 %% 6. Calculate Post-Fault Voltages at all buses
93 %  $V_{post}(i) = V_{prefault}(i) - Zbus(i, fault\_bus) * If_6$ 
94 V_post = zeros(nbus, 1);
95 for i = 1:nbus
96     V_post(i) = V_prefault(i) - Zbus(i, fault_bus) * If_6;
97 end
98
99 %% 7. Display Results
100 fprintf('--- Bolted 3-Phase Fault at Bus %d (Loaded System) ---\n', fault_bus);
101
102 fprintf('--- Post-Fault Voltages ---\n');
103 fprintf('Bus\t Magnitude (pu)\t Angle (deg)\n');
104 for i = 1:nbus
105     fprintf('%d\t %8.4f\t\t %8.4f\n', i, abs(V_post(i)), angle(V_post(i)) * 180/pi);
106 end
107
108 %% 8. Display Summary
109 iter = 4; % NR iterations from load flow
110 Z66 = Zbus(fault_bus, fault_bus);
111
112 fprintf('\n=====\n');
113 fprintf(' SUMMARY\n');
114 fprintf('=====\n');
115 fprintf(' Pre-fault voltages : from NR load flow (%d iterations)\n', iter);

```

```

116 fprintf(' Fault location      : Bus %d (bolted, Zf = 0)\n', fault_bus);
117 fprintf(' Fault current |If|   : %.4f pu\n', abs(If_6));
118 fprintf(' Z66 (Thevenin)         : j%.4f pu\n', imag(Z66));
119 fprintf(' Bus 6 post-fault V       : %.2e pu (~0, bolted) OK\n', abs(V_post(fault_bus)));
120 fprintf('===== \n\n');
121
122 %% 9. Plot: Pre vs Post-fault voltage magnitudes
123 figure('Name','Pre vs Post Fault Voltages','NumberTitle','off','Position',[100 100 680 400]);
124 buses = 1:nbus;
125 V_pre_mag = abs(V_prefault);
126 V_post_mag = abs(V_post);
127
128 bar(buses - 0.2, V_pre_mag, 0.35, 'FaceColor', [0.25 0.55 0.85], 'EdgeColor','none'); hold ←
on;
129 bar(buses + 0.2, V_post_mag, 0.35, 'FaceColor', [0.85 0.40 0.25], 'EdgeColor','none');
130 yline(1.0, '--', 'Color',[0.5 0.5 0.5], 'LineWidth', 1, 'Label', '1.0 pu nominal');
131
132 xlabel('Bus Number', 'FontSize', 12);
133 ylabel('Voltage (pu)', 'FontSize', 12);
134 title('Pre-fault vs Post-fault Voltage Magnitudes - 3-Phase Bolted Fault at Bus 6', 'FontSize'←
', 11);
135 legend('Pre-fault |V|', 'Post-fault |V|', 'Location', 'northwest', 'FontSize', 11);
136 xticks(1:nbus);
137 xlim([0.4, nbus+0.6]);
138 ylim([0, 1.15]);
139 grid on; grid minor;
140 set(gca, 'FontSize', 11);
141
142 % Label bar values
143 for i = 1:nbus
144     text(i-0.2, V_pre_mag(i) + 0.015, sprintf('%.4f', V_pre_mag(i)), 'HorizontalAlignment',←
'center', 'FontSize',9);
145     text(i+0.2, V_post_mag(i) + 0.015, sprintf('%.4f', V_post_mag(i)), 'HorizontalAlignment',←
'center', 'FontSize',9, 'Color',[0.6 0.1 0.0]);
146 end

```

Listing 2: Three-Phase Fault Analysis Implementation